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B.Sc. part-II (Sub). Date - 08-05-2020.
Topic - Maxima and Minima of functions of single variable.

Q.1. Find the maximum and minimum values of

$$2x^3-21x^2+36x-20.$$

Soln. Let $y = 2x^3-21x^2+36x-20$ ——— (I)

Then, Diff. w.r.t. x, we get

$$\frac{dy}{dx} = 6x^2-42x+36 \text{ ——— (II)}$$

Again, diff. w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 12x-42 \text{ ——— III}$$

For maximum or minimum values of y, we have

$$\frac{dy}{dx}=0 \Rightarrow 6x^2-42x+36=0 \Rightarrow x^2-7x+6=0$$

$$\Rightarrow x^2-6x-x+6=0 \Rightarrow x(x-6)-1(x-6)=0 \Rightarrow (x-1)(x-6)=0$$

$\therefore x=1$, or $x=6$.

When $x=1$, then from III, $\frac{d^2y}{dx^2} = 12\times1-42 = 12-42 = -30 < 0$.

Hence, y is maximum, when $x=1$

From (I), the maximum value of $y = 2\times1^3-21\times1^2+36\times1-20$
$= 2-21+36-20$
$= 38-41 = -3$

When $x=6$, then from III, $\frac{d^2y}{dx^2} = 12\times6-42$
$= 12\times6-42$
$= 72-42 = 30 > 0$.

Hence, y is minimum, when $x=6$,

Hence, from (I), the minimum value of y

$= 2\times6^3-21\times6^2+36\times6-20$

$= 2\times6\times6\times6-21\times6\times6+216-20$

$= 432-756+216-20$

$= -128.$

Ans.

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Q.2. Find the maximum value of $x^{\frac{1}{x}}$

Soln: Let $y = x^{\frac{1}{x}}$

Taking logarithm on both sides, we get

$\log y = \log x^{\frac{1}{x}} \Rightarrow \log y = \frac{1}{x}\log x$

$\therefore$ Diff. w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}\log x + \frac{1}{x}\times\frac{1}{x} = \frac{1}{x^2}(1-\log x) \quad \text{—— (I)}$$

For max. or min. of y, $1 = \log x$ $\therefore x = e$

Differentiating (I) w.r.t. x, we get

$$-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{y}\times\frac{d^2y}{dx^2} = \frac{-2}{x^3}(1-\log x) - \frac{1}{x^3}$$

$\therefore$ When $x = e$, $\frac{1}{y}\frac{d^2y}{dx^2} = -\frac{1}{e^3}$

$$\therefore \frac{d^2y}{dx^2} = -e^{\frac{1}{e}}\cdot\frac{1}{e^3} < 0$$

$\therefore y$ is max. when $x = e$ and the max. value $= e^{\frac{1}{e}}$

Q.3. Show that x^x is a minimum for $x = \frac{1}{e}$.

Soln: Let $y = x^x$

Taking logarithm on both sides, we get

$\log y = \log x^x \Rightarrow \log y = x\log x$

If the logarithm of a function is minimum, then the function is also minimum. $\therefore$ Let $u = x\log x$ — I

Diff. w.r.t. x, $\frac{du}{dx} = 1.\log x + x.\frac{1}{x} = 1 + \log x$ — II

Again, diff. w.r.t. x, $\frac{d^2u}{dx^2} = 0 + \frac{1}{x} = \frac{1}{x}$ —— III

For maxima or minima, $\frac{du}{dx} = 0$

i.e. $1 + \log x = 0$, i.e $\log x = -1 = \log e^{-1}$

$\therefore x = \frac{1}{e}$

From III, $\frac{d^2u}{dx^2} = e$ as $x = \frac{1}{e}$

$\therefore u$ is minimum as $\frac{d^2u}{dx^2} > 0$. Hence, the given function y is also minimum when $x = \frac{1}{e}$ Ans.